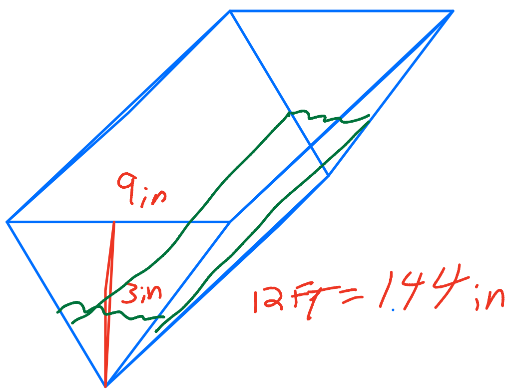
Period 1, Jan 22, 2025

$V = \frac{1}{2} b \cdot h \cdot \underline{144}$

$b = 3h$



$\frac{dV}{dT} = ?$

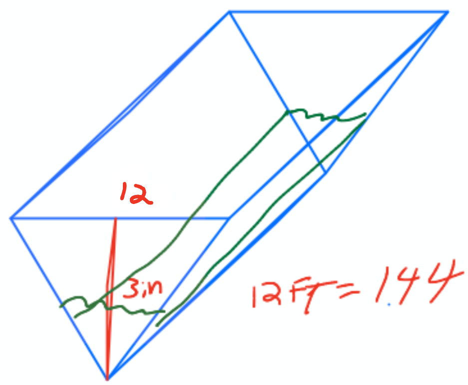
$\frac{dh}{dT} = \frac{1}{4}$ in/min $\quad h = 2$

$V = \frac{1}{2} \cdot 3h \cdot h \cdot 144$

$V = 216 h^2$

$\frac{dV}{dT} = 432 h \frac{dh}{dT} = 432 \cdot 2 \cdot \frac{1}{4}$

$\frac{dV}{dT} = 216$ in^3/min



12ft = 144 in

$$\frac{dh}{dT} = \frac{1}{4} \text{ in/min}$$

$$\frac{dV}{dT} = ? \qquad h = 2$$

$$4h = B \qquad V = \frac{1}{2} B \cdot h \cdot 144$$

$$V = \frac{1}{2} \cdot 4h \cdot h \cdot 144$$

$$V = 288h^2$$

$$\frac{dV}{dT} = 576\, h \frac{dh}{dT} = 576 \cdot 2 \cdot \frac{1}{4} = 288 \text{ in}^3/\text{min}$$

---

1 hour Later

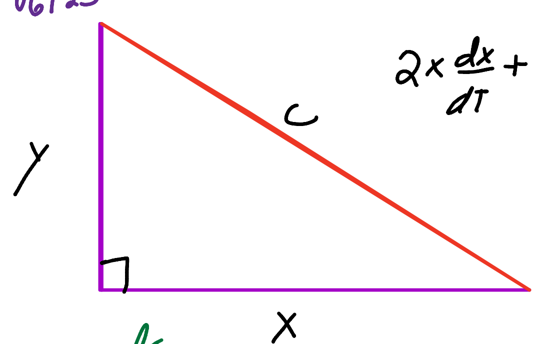
$y = 45$ miles

$x = 70$ miles

$$45^2 + 70^2 = c^2$$

$$\sqrt{6925} = c$$

$$x^2 + y^2 = c^2$$



45mph

$$\frac{dy}{dT} = 45 \text{ mph}$$

$$2x\frac{dx}{dT} + 2y\frac{dy}{dT} = 2c\frac{dc}{dT}$$

$$\frac{dx}{dT} = 70 \text{ mph}$$

$$\frac{dc}{dT} = \frac{6925}{\sqrt{6925}} \cdot \frac{\sqrt{6925}}{\sqrt{6925}}$$

$$\frac{dc}{dT} = \frac{6925\sqrt{6925}}{6925}$$

$$2(70)(70) + 2(45)(45) = 2 \cdot c \cdot \frac{dc}{dT}$$

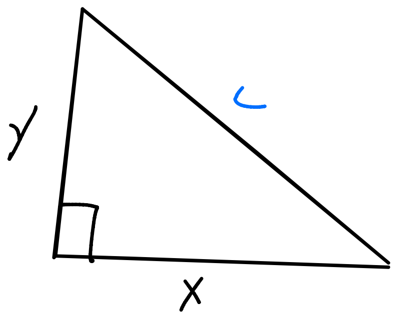
$$9800 + 4050$$

$$\frac{\cancel{13850}^{6925}}{2\sqrt{6925}} = \frac{\cancel{2} \cdot \cancel{\sqrt{6925}}}{\cancel{2\sqrt{6925}}} \frac{dc}{dT}$$

$$\frac{dc}{dT} = \sqrt{6925} = 5\sqrt{277} \text{ mph}$$

after 1hour
$y = 45$ miles
$x = 65$ miles

$\frac{dy}{dT} = 45$ mph



$\frac{dx}{dT} = 65$ mph

$x^2 + y^2 = c^2$

$2x\frac{dx}{dT} + 2y\frac{dy}{dT} = 2c\frac{dc}{dT}$

$45^2 + 65^2 = c^2$

$\sqrt{6250} = c$

$\underbrace{2 \cdot 65 \cdot 65 + 2 \cdot 45 \cdot 45}_{12500} = 2c\frac{dc}{dT}$

$\frac{12500}{2\sqrt{6250}} = \frac{\cancel{2\sqrt{6250}}\frac{dc}{dT}}{\cancel{2\sqrt{6250}}}$

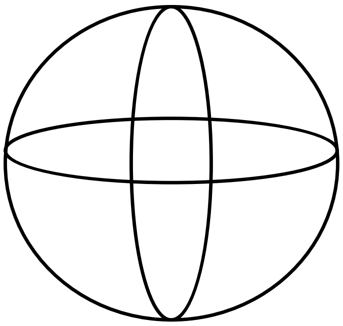
$\sqrt{625} = 25$

$\sqrt{6250} = \frac{dc}{dT}$

$625 \cdot 10$

$\sqrt{6250} = \frac{250\sqrt{10}}{10} = 25\sqrt{10}$

⑥



$\frac{dV}{dT} = 650 \ cm^3/min$

$\frac{dr}{dT}$ when $r = 16cm$

$650 = 4 \cdot \pi \cdot 16^2 \frac{dr}{dT}$

$\frac{650}{1024\pi} = \frac{dr}{dT}$

$\frac{325}{512\pi} cm/min = \frac{dr}{dT}$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dT} = 4\pi r^2 \frac{dr}{dT}$

$\frac{dV}{dT} = 650 \ cm^3/min$

$\frac{dr}{dT}$ when $r = 15cm$

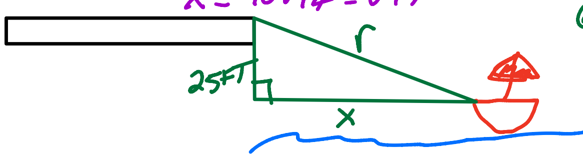
$650 = 4\pi \ 15^2 \frac{dr}{dT}$

$\frac{650}{900\pi} cm/min = \frac{dr}{dT}$

$\frac{13}{18\pi} = \frac{dr}{dT}$

$25^2 + x^2 = 45^2$

$x = 10\sqrt{14} = \sqrt{1400}$



$25^2 + x^2 = r^2$

$625 + x^2 = r^2$

$2x \frac{dx}{dT} = 2r \frac{dr}{dT}$

$\frac{dr}{dT} = -14 \ FT/s$

when $r = 45$ what is $\frac{dx}{dT}$

$2 \cdot 10\sqrt{14} \cdot \frac{dx}{dT} = 2 \cdot 45 \cdot (-14 \ FT/s)$

$\frac{dx}{dT} = \frac{-630}{10\sqrt{14}}$

$\frac{dr}{dT} = -10 \ FT/s$ when $r = 45$ what is $\frac{dx}{dT}$

$2 \cdot 10\sqrt{14} \frac{dx}{dT} = 2 \cdot 45 \cdot (-10 \ FT/sec)$

$\frac{dx}{dT} = \frac{-45}{\sqrt{14}} \ FT/sec$

⑧

$$\frac{dy}{dT}$$

$$\frac{x}{y} = \frac{a}{8}$$

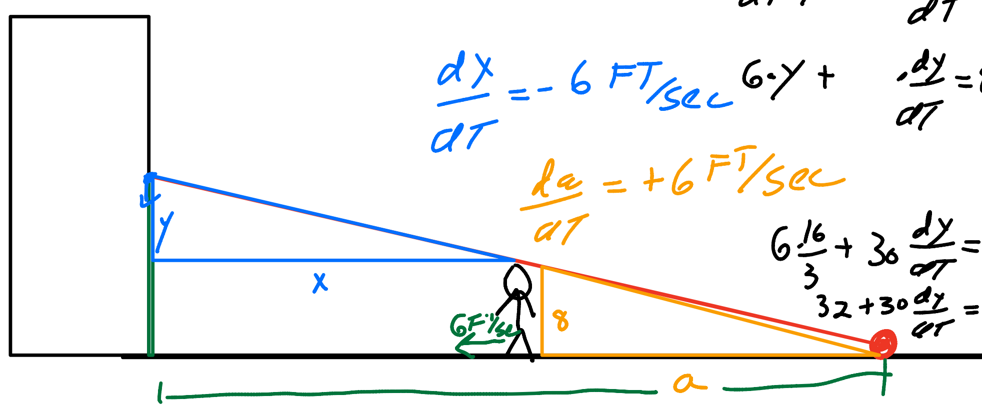
$$ay = 8x$$

$$\frac{da}{dT} \cdot y + a \cdot \frac{dy}{dT} = 8\frac{dx}{dT}$$

$$\frac{dx}{dT} = -6 \text{ FT/sec}$$

$$6 \cdot y + \cdot \frac{dy}{dT} = 8 \cdot -6 \frac{\text{FT}}{\text{sec}}$$

$$\frac{da}{dT} = +6 \text{ FT/sec}$$



$$6\frac{16}{3} + 30\frac{dy}{dT} = -48\frac{\text{FT}}{\text{sec}}$$

$$32 + 30\frac{dy}{dT} = -48\frac{\text{FT}}{\text{s}}$$

$X = 20$

$a = 30$

$$30\frac{dy}{dT} = -80 \frac{\text{FT}}{\text{sec}}$$

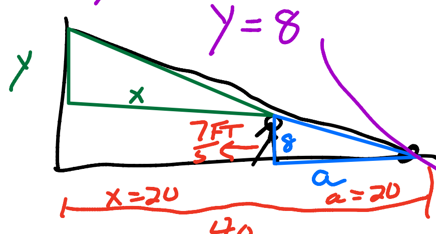
$$\frac{dy}{dT} = \frac{-8}{3} \text{FT/sec}$$

$$\frac{20}{y} = \frac{30}{8}$$

$$30y = 8 \cdot 20 \Rightarrow y = \frac{16}{3}$$

$$\frac{20}{y} = \frac{20}{8}$$

$y = 8$



$$\frac{dx}{dT} = -7 \text{ FT/sec}$$

$$\frac{da}{dT} = +7 \text{ FT/sec}$$

$$\frac{x}{y} = \frac{a}{8}$$

$$8x = ay$$

$$8\frac{dx}{dT} = y\frac{da}{dT} + a \cdot \frac{dy}{dT}$$

$$8 \cdot -7 \text{ FT/sec} = y \cdot \frac{7\text{FT}}{\text{sec}} + 20\frac{dy}{dT}$$

$$-56 = 8 \cdot 7 + 20\frac{dy}{dT}$$

$$-56 = 56 + 20\frac{dy}{dT} \Rightarrow \frac{dy}{dT} = \frac{-112}{20} \frac{\text{FT}}{\text{sec}}$$

Find $\frac{dy}{dT}$ when $x = 20$ FT

c) $\frac{dy}{dx} = x^{1/3} + x\sqrt{x} - 2$; when $x = 1, y = 2$

$\frac{1}{3} + 1 = \frac{4}{3}$

$\frac{3}{2} + 1 = \frac{5}{2}$

$$\frac{dy}{dx} = x^{\frac{1}{3}} + x^{\frac{3}{2}} - 2$$

$$\int dy = \int (x^{\frac{1}{3}} + x^{\frac{3}{2}} - 2)\,dx$$

$$y = 1 \cdot \frac{3}{4} \cdot x^{4/3} + 1 \cdot \frac{2}{5} x^{5/2} - 2x + C$$

$$2 = \frac{3}{4}(1)^{4/3} + \frac{2}{5}(1)^{5/2} \cancel{-2(1)} + C$$

$+2 \quad +2$

$$4 = \frac{3 \cdot 5}{4 \cdot 5} + \frac{2 \cdot 4}{5 \cdot 4} + C$$

$$4 = \frac{15 + 8}{20} + C$$

$$4 = \frac{23}{20} + C$$

$-\frac{23}{20} \quad -\frac{23}{20}$

$$4 - 1\frac{3}{20} = 2\frac{17}{20} = \frac{57}{20}$$

d) $f'(x) = x - 2\sin x\,;\ f(\pi) = 0$

$$\int (x - 2\sin x)\,dx$$

$$F(x) = 1 \cdot \tfrac{1}{2} x^{1+1=2} - 2(-\cos x) + C$$

$$F(x) = \tfrac{1}{2}x^2 + 2\cos x + C$$

$$0 = \tfrac{1}{2}\pi^2 + 2\cos\pi + C$$

$$0 = \frac{\pi^2}{2} - 2 + C$$

$$2 - \frac{\pi^2}{2} = C$$

---

e) $\frac{d^2y}{dx^2} = e^x$; when $x = 0, y = 2,$ when $x = 1, y = e$

$$\frac{dy}{dx} = \int e^x\,dx$$

$$\frac{dy}{dx} = e^x + C_1$$

$$y = \int (e^x + C_1)\,dx$$

$$y = e^x + C_1 \cdot x + C_2$$

$$2 = e^0 + C_1 \cdot 0 + C_2$$

$$2 = 1 + 0 + C_2$$

$$1 = C_2$$

$$y = e^x + C_1 x + 1$$

$$e = \cancel{e^1} + C_1(1) + 1$$

$$-e \quad -\cancel{e}$$

$$0 = C_1 + 1$$

$$C_1 = -1$$

$$y = e^x + -1x + 1$$

3. A particle in rectilinear motion moves along the x-axis with velocity $v(t) = t^2 + t$, $t \geq 0$. If the particle is at $x = -1$ when $t = 0$, then what is the position x of the particle at time $t = 3$?
(Hint: $s(t)$ and $x(t)$ are *both* commonly used to symbolize position as a function of time, t)

$$V(T) = T^2 + T$$

$$S(T) = \int (T^2 + T)\, dT = \frac{1}{3}T^3 + \frac{1}{2}T^2 + C$$

$$-1 = \frac{1}{3}(0)^3 + \frac{1}{2}(0)^2 + C$$

$$-1 = C$$

$$S(T) = X(T) = \frac{1}{3}T^3 + \frac{1}{2}T^2 - 1$$

$$S(3) = X(3) = \frac{1}{3}(3)^3 + \frac{1}{2}(3)^2 - 1 = 9 + \frac{9}{2} - 1 = 8 + \frac{9}{2} = 8 + 4\frac{1}{2}$$

$$12\frac{1}{2}$$

4. Given the acceleration function below, use Calculus techniques to find the distance s of the object from the origin under the initial conditions, $s(0) = 0\ ft$ and $v(0) = 5$ ft/s

$a(t) = \sin t\ ft/s^2$

$$\int a(T) = V(T)$$

$$\int \sin T\, dT = -\cos T + C = V(T)$$

$$-\cos 0 + C = 5$$

$$-1 + c = 5$$

$$c = 6$$

$$V(T) = -\cos T + 6$$

$$S(T) = \int V(T)\, dT = \int (-\cos T + 6)\, dT = -\sin T + 6T + C_2$$

$$S(T) = -\sin T + 6T$$

$$0 = -\sin 0 + 6(0) + C_2$$

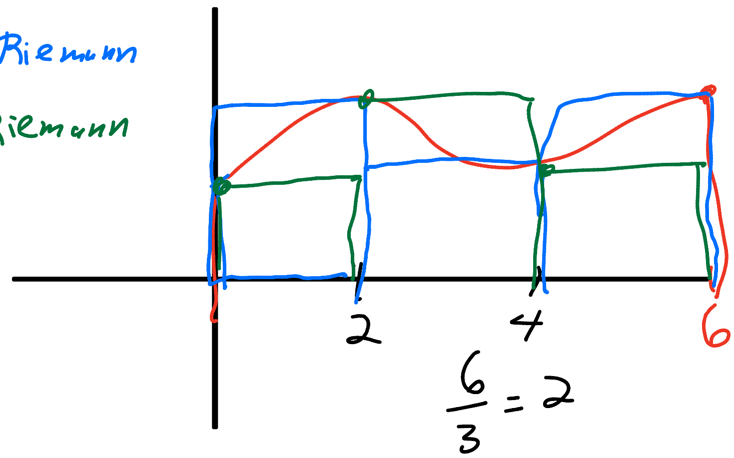
$$0 = 0 + 0 + C_2$$

$$0 = C_2$$

Using Riemann Sum to Approximate Area

Right Riemann

Left Riemann



$n=3$

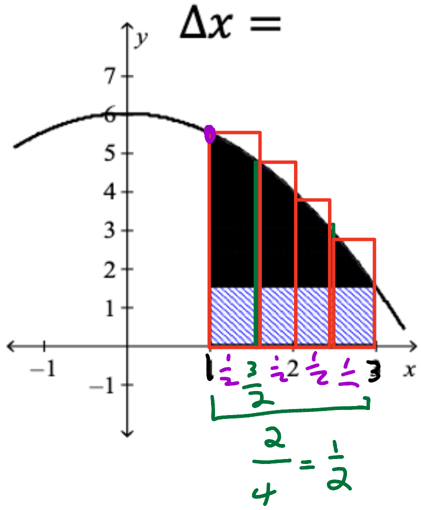
what is The area under curve

Example 1: Use a Left Riemann Sum with **four** equal rectangles to *approximate* of the area of the region lying between the graph of $f(x) = \left(\frac{-1}{2}\right)x^2 + 6$ and the x-axis between $x = 1$ and $x = 3$.

1) Area $\approx$ $\frac{1}{2}(5\frac{1}{2})$ + $\frac{1}{2}(4\frac{7}{8})$ + $\frac{1}{2}(4)$ + $\frac{1}{2}(2\frac{7}{8})$

2) The base is 8.625. So the sub intervals are:

$\Delta x =$



[1,1.5], [1.5,2], [2,2.5], [2.5,3]

3) To find the first approximation use the **left endpoints** as the heights of your rectangles.

$-\frac{1}{2}\left(\frac{3}{2}\right)^2 + 6$

$-\frac{9}{8} + 6$

x	$-\frac{1}{2}x^2+6$
1	$5\frac{1}{2}$
1.5	$4\frac{7}{8}$
2	4
2.5	$2\frac{7}{8}$

$-\frac{1}{2}\left(\frac{5}{2}\right)^2 + 6$

$-\frac{25}{8} + 6$

$-3\frac{1}{8} + 6$

$$y = \int_1^3 \left(-\tfrac{1}{2}x^2 + 6\right) dx = -\frac{1}{6}x^3 + 6x + c \Big]_1^3 =$$

$$-\frac{1}{6}(3)^3 + 6(3) + c - \left[-\frac{1}{6}(1)^3 + 6(1) + c\right]$$

$$-\frac{9}{2} + 18 + \cancel{c} + \frac{1}{6} - 6 - \cancel{c}$$

$$-\frac{9}{2} + 18 + \frac{1}{6} + -6 = 12 - \frac{27}{6} + \frac{1}{6}$$

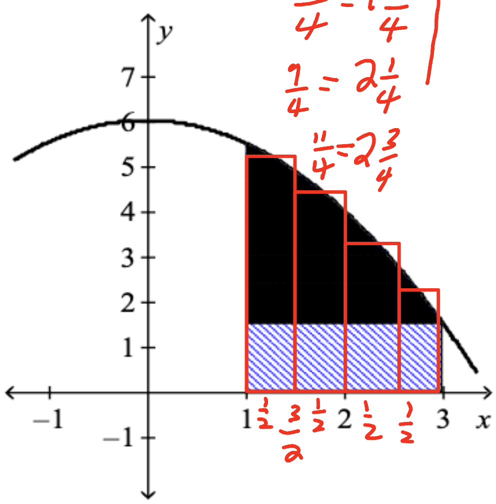
$$12 - \frac{26}{6}$$

$$12 - 4\frac{1}{3} = 8 - \frac{1}{3}$$

$$7\frac{2}{3}$$

Example 2: Use a Midpoint Riemann Sum with **four** rectangles to *approximate* of the area of the region lying between the graph of $f(x) = \left(\frac{-1}{2}\right)x^2 + 6$ and the x-axis between $x = 1$ and $x = 3$.

X	Y
$\frac{5}{4} = 1\frac{1}{4}$	
$\frac{7}{4} = 1\frac{3}{4}$	
$\frac{9}{4} = 2\frac{1}{4}$	
$\frac{11}{4} = 2\frac{3}{4}$	



$-\frac{1}{2}\left(\frac{5}{4}\right)^2 + \frac{192}{32} = \frac{-25}{32} + \frac{192}{32} = \frac{167}{32}$

$-\frac{1}{2}\left(\frac{7}{4}\right)^2 + \frac{192}{32} = -\frac{49}{32} + \frac{192}{32} = \frac{143}{32}$

$-\frac{1}{2}\left(\frac{9}{4}\right)^2 + \frac{192}{32} = \frac{-81}{32} + \frac{192}{32} = \frac{111}{32}$

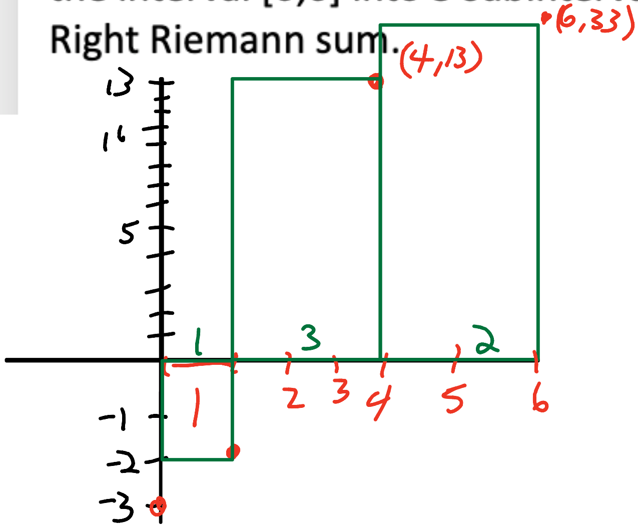
$-\frac{1}{2}\left(\frac{11}{4}\right)^2 + \frac{192}{32} = \frac{-121}{32} + \frac{192}{32} = \frac{71}{32}$

$\frac{1}{2}\left(\frac{167}{32}\right) + \frac{1}{2}\left(\frac{143}{32}\right) + \frac{1}{2}\left(\frac{111}{32}\right) + \frac{71}{32}\cdot\frac{1}{2} =$

$\frac{492}{64} = 7.6875$

For MC (simplify all the way): 7.6875

Example 3: For the function $f(x) = x^2 - 3$, $0 \le x \le 6$, partition the interval [0,6] into 3 subintervals [0,1],[1,4],[4,6] and form the Right Riemann sum.



$F(0) = -3$

$F(1) = -2$

$F(4) = 4^2 - 3 = 13$

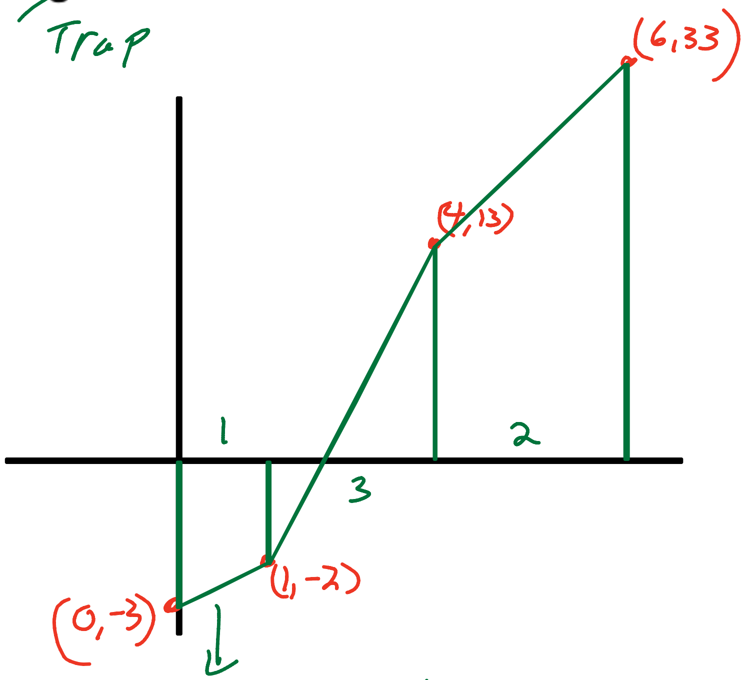
$F(6) = 6^2 - 3 = 33$

$1\cdot -2 + 3\cdot 13 + 2\cdot 33$

$-2 + 39 + 66 = 103$

Example 3: For the function $f(x) = x^2 - 3$, $0 \leq x \leq 6$, partition the interval [0,6] into 3 subintervals [0,1],[1,4],[4,6] and form the ~~Right~~ Riemann sum.

Trap



$$\frac{1}{2}(-3+-2)1+\frac{1}{2}(-2+13)\cdot 3+\frac{1}{2}(13+33)\cdot 2$$

$$-\frac{5}{2}+\frac{33}{2}+\frac{92}{2}=\frac{126}{2}=60$$

$$\int_0^6 (x^2-3)\,dx = \frac{1}{3}x^3-3x+C \Big[_0^6$$

$$\frac{1}{3}6^3-3(6)+\cancel{C}-\cancel{\left[\frac{1}{3}(0)^3-3(0)+C\right]}$$

$$\frac{216}{3}-18=54$$